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II

Concerning the ultimate foundation of the differentiation of regions in space

of position. Region really consists rather in the relation of the system of these positions to absolute space. The position of the parts of any extended object, with respect to each other, can be sufficiently recognised from the object itself. The region, however, to which this order of the parts is directed, is related to space outside, but not with reference to its localities, for this would be nothing else than the position of just those parts in an external relation; region is related rather to space in general as a unity, of which each extension must be regarded as a part. It is no wonder if the reader finds these concepts still very incomprehensible; but they should become clear in due course. I add, therefore, nothing further, except that my intention in this paper is to see whether, in the intuitive judgements of extension, such as include geometry, a clear proof can be found that *absolute space has its own reality independently of the existence of all matter and that it is itself the ultimate foundation of the possibility of its composition*. Everyone knows how futile the efforts of philosophers have been to place this point once and for all beyond dispute, by means of the most abstract judgements of metaphysics. I know of no attempt to execute this a posteriori (namely, by using other undeniable propositions, themselves lying outside the realm of metaphysics, but able, when applied in particular concrete cases, to offer a touchstone of their correctness), apart from the treatise of the distinguished *Euler the Elder*⁵ in the history of the Royal Academy of Sciences in Berlin for the year 1748. It did not, however, quite fulfil its purpose, since it only shows the difficulties of giving a definite significance to the most general laws of motion, when the only concept of space that is accepted is that which is derived from the abstraction from the relation of real things. But it leaves untouched the not less significant difficulties which remain, when the supposed laws are applied, when one wishes to represent them according to the concept of absolute space, in a particular concrete case. The proof which I am seeking here is intended to place in the hands, not of engineers, as was the intention

377 The illustrious *Leibniz* enriched various departments of knowledge with many genuine insights. But the world waited in vain for him to execute projects far greater still. Whether the reason was that his efforts seemed too incomplete to him,—a reservation peculiar to men of distinction, that has continually deprived learning of many valuable fragments,—or whether it was with *Leibniz*, as *Boerhaave*¹ suspects it was with great chemists: that they often claimed the ability to perform certain undertakings, as if they possessed the ability, whereas, in reality, they possessed only the conviction and trust in their own skill, that once they wished to attempt the performance of an undertaking, they could not but be successful: I do not wish to decide here what the explanation is. At least it looks as if a certain mathematical discipline, which he entitled in advance 'Analysis situs', the loss of which *Buffon*,² in considering the natural folding together in seeds³ lamented, was probably never anything more than a thing of the imagination.⁴ I do not know how far the object, which I propose to examine here, is related to that which the great man had in mind. To judge from the meaning of words alone, I am engaged in a philosophic search for the ultimate foundation of the possibility of that, of which *Leibniz* intended to determine the magnitudes mathematically. For the positions of the parts of space, in relation to each other, presuppose the region, according to which they are ordered in such a relation. In the most abstract sense, region does not consist of the relation of one thing in space to the next. That would really be the concept

of Herr *Euler*, but in the hands of geometers themselves a convincing proof that would enable them to assert, with the clearness customary to them, the reality of their absolute space. For this purpose, I make the following preparation.

Because of its three dimensions, three surfaces can be conceived in physical space. They all intersect each other at right angles. Since we know nothing external to us through the senses, except in so far as it stands in relation to ourselves, it is no wonder that we derive from the relation of these intersecting surfaces to our body the ultimate foundation of generating the concept of regions in space. The surface on which the length of our body stands vertically is called, with respect to ourselves, horizontal; and this horizontal surface gives occasion for the differentiation of objects which we indicate by *above* and *below*. Two other surfaces can stand vertically on this surface and they can, at the same time, intersect each other at right angles, so that the length of the human body is conceived along the line of the intersection. One of these vertical surfaces divides the body into two externally similar halves and gives the foundation of the distinction between the *right* and the *left* half; the other vertical surface which stands perpendicularly to it, enables us to conceive the *front* and *back* side. In a sheet of writing for example, we distinguish the upper from the lower part of the writing; we notice the difference between the front and the back side; and then we notice the position of the written characters from left to right, or vice versa. Turn the sheet how one will, the parts which are ordered on the surface always have the same position here with respect to each other, and the figure is, in all parts, one and the same. But by this representation, the distinction of regions comes so much into consideration and is so closely connected with the impression made by the visible object that the very same piece of writing becomes unrecognisable, when it is seen with everything turned from the right to the left, which before had the opposite position.

Even our judgements on terrestrial regions are subordinated to the concept we have of regions in general, in so far as they are determined, in relation to the sides of our bodies. Whatsoever relations we otherwise recognise in the heavens and on the earth, independently of this fundamental concept, are merely the positions of objects in relation to each other. No matter how well I know the order of the parts of the horizon, I can only determine the regions, in accordance with this knowledge, if I am aware of the direction in which the order runs. The most accurate of heavenly charts, no matter how accurately I have it in mind, would not in the end enable me to know from the known region, for example from the north, on which side of the horizon I should seek the rising sun, if, apart from the position of the stars to each other, the regions were not determined by the position of the sketch in relation to my hands. The same holds true of geographical, indeed of our most ordinary knowledge of the position of places; such knowledge is of no help to us, so long as we are unable to place the so ordered things and the whole system of reciprocally related positions, according to regions, through the relation to the sides of our bodies. There even exists a very noted characteristic of the products of nature, which can itself now and then give occasion to the distinction of kinds, in the definite region where the order of their parts is reversed, and whereby two creatures can be distinguished, even though, in respect both of size and proportion and even of the situation of the parts relative to each other, they may be in perfect agreement.⁶ The hair on the crown of the head of all human beings is directed from the left to the right hand side. All hops wind round their poles from left to right; beans, however, twist in the opposite direction. Almost all snails, with the exception of perhaps three species, coil from the left side to the right, looking down from above, that is from the point of the shell to the mouth. This definite quality is immutably present in exactly the same species, without any relation to the hemisphere where they are to be found, or to the direction of the daily

movement of the sun and moon which, with us runs from left to right, but which for those living in the Antipodes runs from right to left. This is because, in the natural generations mentioned, the cause of the convolutions lies in the seeds themselves. On the other hand, where a certain turning can be attributed to the course of the heavenly bodies, as for example the law *Mariotte*⁷ claims to have observed in the case of the winds, which readily run through the whole compass from left to right from new moon to full moon, then these circular movements must run in the opposite direction in the other hemisphere, as indeed *Don Ulloa*⁸ really thinks he has found confirmed by his observations in the southern ocean.

Since the distinct feeling of the right and the left side is of such great necessity to the judgement of the regions, nature has at the same time attached it to the mechanical structure of the human body. By its means one side, namely the right hand one, has an undoubted superiority in skill, and, perhaps, also in strength, over the left. Hence all the peoples of the earth are right handed (leaving aside individual exceptions which, like that of being cross-eyed, cannot upset the universality of the rule, according to the natural order). One moves one's body more easily from the right to the left than in the opposite direction when one mounts a horse or steps over a pit. Everywhere one writes with the right hand and one does everything with it, for which skill or strength is required. However, just as the right side seems to have the advantage in mobile power, so the left side has the advantage over the right side in respect of sensitivity, if certain scientists are to be believed—for example *Borelli*⁹ and *Bonnet*.¹⁰ The former asserts of the left eye and the latter of the left ear that the sense in them is stronger than that in the identically named organ on the right side. And thus it is that both sides of the human body, irrespective of their great external similarity, are sufficiently distinguished, by means of clear sensation, leaving aside the differing situation of the internal parts and the perceptible beating of the heart,

since this muscle in its continual contraction touches, in oblique motion, the left side of the breast with its tip.

We wish, therefore, to show that the complete principle of determining a physical form does not rest merely on the relation and the situation of the parts, with respect to each other, but also on its relation to general absolute space, as conceived by geometers; indeed, in such a way that this relation cannot be immediately perceived, though, perhaps, the physical differences that rest uniquely and alone on this ground can be. When two figures, drawn on a flat surface, are like and similar, they cover each other. But it is often different with physical extension or even with lines and surfaces not lying on a flat surface. They can be perfectly like and similar and yet be in themselves so different that the limits of the one cannot at the same time be the limits of the other. The thread of a screw which goes round its pin from left to right will never fit into a nut where the thread runs from right to left, even though the size of the pin and the number of the screw-turns are the same. A spherical triangle can be perfectly like and similar to another without however covering it. But the most common and the clearest example is to be found in the members of the human body, which are ordered symmetrically with respect to the vertical surface. The right hand is similar to and like the left hand, and merely looking at one of them, at the proportion and the situation of the parts to each other, and at the size of the whole, a complete description of the one must apply, in all respects, to the other.

An object which is completely like and similar to another, although it cannot be included exactly within the same limits, I call its *incongruent counterpart*. In order to demonstrate the possibility of an incongruent counterpart a body is taken which does not consist of two halves arranged symmetrically with reference to a single intersecting surface but rather, for example, a *human hand*. From all points of its surface one extends perpendicular lines to a board placed opposite the object. One extends these lines exactly so far

behind the board as the points lie before it. When the end points of these so extended lines are connected, they constitute the surface of a bodily figure, which is the incongruent counterpart of the original object. That is, when the given hand is a right hand, its counterpart is a left hand. The reflection of an object in a mirror rests on exactly the same principles. For the object appears always exactly so far behind the mirror as it stands before its surface. Thus the image of a right hand is always a left hand in the mirror. Should the object itself consist of two incongruent counterparts as, for example, does the human body when it is divided from back to front by means of a vertical intersection, then its image is congruent to it. This can be easily seen when one imagines it turned a half circle. For the counterpart of an object's counterpart is, of necessity, congruent to that object.

So much may be sufficient to understand the possibility of completely like and similar and yet incongruent spaces. We turn now to the philosophical application of these concepts. It is already clear from the everyday example of the two hands that the figure of a body can be completely similar to that of another, and that the size of the extension can be, in both, exactly the same; and that yet, however, an internal difference remains: namely, that the surface that includes the one could not possibly include the other. As the surface limiting the bodily space of the one cannot serve as a limit for the other, twist and turn it how one will, this difference must, therefore, be such as rests on an inner principle. This inner principle of difference cannot, however, be connected with the different way in which the parts of the body are connected with each other. For, as one sees from the given example, everything can be perfectly identical in this respect. Let it be imagined that the first created thing were a human hand, then it must necessarily be either a right hand or a left hand. In order to produce the one a different action of the creative cause is necessary from that, by means of which its counterpart could be produced.

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If one accepts the concept of modern, in particular, German philosophers, that space only consists of the external relations of parts of matter, which exist alongside one another, then all real space would be, in the example used, simply that *which this hand takes up*. However, since there is no difference in the relations of the parts to each other, whether right hand or left, the hand would be completely indeterminate with respect to such a quality, that is, it would fit on either side of the human body. But that is impossible.

From this it is clear that the determinations of space are not consequences of the situations of the parts of matter relative to each other; rather are the latter consequences of the former. It is also clear that in the constitution of bodies differences, and real differences at that, can be found; and these differences are connected purely with *absolute and original space*, for it is only through it that the relation of physical things is possible. It is also clear that since absolute space is not an object of external sensation, but rather a fundamental concept, which makes all these sensations possible in the first place, we can only perceive through the relation to other bodies that which, in the form of a body, purely concerns its relation to pure space.

A reflective reader will therefore regard the concept of space in the way geometers regard it, and also as perceptive philosophers have taken it up into the theory of natural science, as other than a mere entity of reason.¹¹ Nonetheless, there is no lack of difficulties surrounding the concept when one tries to grasp with the ideas of reason its reality, evident enough to the inner sense. But this difficulty appears everywhere, if one still wishes to philosophise about the first data of our experience. But this difficulty is never so decisive as that which emerges, when the consequences of an accepted concept contradict the clearest experience.

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